

KLEIN PARADOX FOR OPTICAL SCATTERING FROM EXCITED TARGETS

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The well known Klein paradox for the relativistic Dirac wave equation consists in the computation of possible “negative probabilities” induced by certain potentials in some regimes of energy. The paradox may be resolved employing the notion of electron-positron pair production in which the number of electrons present in a process can increase. The Klein paradox also exists in Maxwell’s equations viewed as the wave equation for photons. In a medium containing “inverted energy populations” of excited atoms, e.g. in a LASER medium, one may again compute possible “negative probabilities”. The resolution of the electromagnetic Klein paradox is that when the atoms decay, the final state may contain more photons than were contained in the initial state. The optical theorem total cross section for scattering photons from excited state atoms may then be computed as negative within a frequency band with matter induced amplification.

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I. INTRODUCTION

At a time in which the properties of relativistic electrons (implicit in the Dirac equation[1] for the spinor wave function) were not so well understood, Klein predicted a “paradox” associated with the problem of reflecting an electron off a step potential. The reflection coefficient R and the transmission coefficient P (induced by the step potential) obey an expected sum rule

$$R + P = 1 \quad (\text{exact}). \quad (1)$$

However, for some step potential heights and for some incident energies

$$R > 1 \quad (\text{possible}). \quad (2)$$

Eqs.(1) and (2) imply the possibility of a negative transmission “probability” coefficient; i.e. $P < 0$ is possible and therein lies the paradox.

The Dirac wave equation for electrons and positrons allows for a physical picture which clarifies the meaning of the Klein paradox[2] and thus removes it[3, 4, 5, 6] as a possible objection to the Dirac theory. When an electron hits the step potential, it is possible to create an electron-positron pair. For such an event, two electrons may be reflected from the step potential when only one electron was incident. The added positron may then be transmitted into the step potential. Such an event, if sufficiently probable, may yield $R > 1$ since more electrons may be reflected from the step potential than are incident. Furthermore, $P < 0$ may be interpreted to mean that the current of the transmitted positron wave is directed oppositely to that which would have been carried by an electron wave moving at the same velocity.

Our purpose is to point out that the Klein paradox[7, 8, 9] is intrinsic to relativistic particles[10, 11, 12, 13] and in particular applies to the photon. For the electromagnetic case, the wave equations for the photon in space-time are merely Maxwell’s equations. To understand the nature of the Klein paradox for photons we may recall the classical results discussed by Rayleigh. Suppose a spherically symmetric target with a fluctuating dipole moment which scatters an electromagnetic wave. We suppose that the electric dipole moment $\mathbf{p}(t)$ response to an applied electric field

$$\mathbf{E}(t) = \Re \{ \mathbf{E}_0 e^{-i\zeta t} \} \quad \text{with} \quad \Im m(\zeta) > 0, \quad (3)$$

is given by

$$\mathbf{p}(t) = \Re \{ \alpha(\zeta) \mathbf{E}_0 e^{-i\zeta t} \}. \quad (4)$$

Here $\alpha(\zeta)$ is the target polarizability.

Rayleigh asserted that the elastic scattering amplitude for an incident electromagnetic wave onto the polarizable target is given in the dipole approximation by

$$F_{i \rightarrow f} = \left(\frac{\omega}{c}\right)^2 \mathbf{e}_f^* \cdot \mathbf{e}_i \alpha(\omega + i0^+) \quad (5)$$

where \mathbf{e}_i and \mathbf{e}_f are, respectively, the initial and final polarization vectors. Averaging over initial polarization and summing over final polarizations yields the elastic differential cross section

$$\begin{aligned} \left(\frac{d\sigma_{el}}{d\Omega}\right) &= \frac{1}{2} \sum_i \sum_f |F_{i \rightarrow f}|^2 \\ &= \frac{1}{2} (1 + \cos^2 \theta) \left(\frac{\omega}{c}\right)^4 |\alpha(\omega + i0^+)|^2, \end{aligned} \quad (6)$$

which implies for the elastic cross section $\sigma_{el} = \int d\sigma_{el}$ that

$$\sigma_{el}(\omega) = \left(\frac{8\pi}{3}\right) \left(\frac{\omega}{c}\right)^4 |\alpha(\omega + i0^+)|^2. \quad (7)$$

For understanding the Klein paradox for electromagnetic waves it is more important to discuss the total cross section which involves the optical theorem

$$\sigma_{tot} = \left(\frac{4\pi c}{\omega}\right) \Im m(F_{i \rightarrow i}) \quad (8)$$

in the form

$$\sigma_{tot}(\omega) = \left(\frac{4\pi\omega}{c}\right) \Im m[\alpha(\omega + i0^+)]. \quad (9)$$

The expected sum rule is that

$$\sigma_{tot}(\omega) = \sigma_{el}(\omega) + \sigma_{in}(\omega), \quad (10)$$

where $\sigma_{in}(\omega)$ is the inelastic cross section. In the commonly studied case in which the target “absorbs” radiation one has a positive total cross section $\sigma_{tot}(\omega) > 0$ since

$$\omega \Im m[\alpha(\omega + i0^+)] > 0 \quad (\text{absorption band}). \quad (11)$$

On the other hand, for a target in some excited energy state, (say) with atoms having “inverted” energy populations, there will exist frequency bands in which

$$\omega \Im m[\alpha(\omega + i0^+)] < 0 \quad (\text{amplifier band}). \quad (12)$$

The optical theorem in the form of Eqs.(9) and (12) yield the following.

The Optical Klein Paradox: If \mathcal{B} is the set of frequencies within which the target is an electromagnetic amplifier, then

$$\sigma_{tot}(\omega \in \mathcal{B}) < 0. \quad (13)$$

A negative total cross section in an amplifying frequency band is only at first glance an impossibility. The purpose of this work is to discuss physical meaning of the optical Klein Paradox for amplifiers such as inverted population targets, e.g. “pumped” LASER materials[14]. For the case of the Dirac equation, pair production allowed (without changing total charge) for more electrons in an outgoing state than were present in the incoming state. This simple fact makes transparent the notion of a negative forward transmission coefficient $P < 0$. Similarly, for an excited state target one incident photon can give rise to two photons in the outgoing state when the excited state target is induced to decay (say) into a target ground state. In an electromagnetic scattering experiment, there can be more radiation behind the target than that which would exist if the excited radiating target were removed.

In Sec.II, we consider an electromagnetic wave traveling through a medium with a dielectric response function $\varepsilon(\zeta)$. For a dilute density per unit volume n of polarizable targets in the medium, the dielectric response

$$\varepsilon(\zeta) = 1 + 4\pi n\alpha(\zeta) + \dots \quad \text{if } |4\pi n\alpha(\zeta)| \ll 1, \quad (14)$$

so that

$$\Im[\varepsilon(\omega + i0^+)] = 4\pi n \Im[\alpha(\omega + i0^+)]. \quad (15)$$

The properties of $\varepsilon(\omega + i0^+)$ within an amplifying frequency band \mathcal{B} will be explored. In Sec.III, the concept of a negative noise temperature T_n will be defined. The definition of an amplifying target will be related to the notion of negative noise temperature T_n [15, 16]. It will be shown that an amplifying frequency band \mathcal{B} may equally well be defined by the notion of a negative noise temperature $T_n(\omega \in \mathcal{B}) < 0$. The electromagnetic Klein paradox occurs in all material systems exhibiting a negative radiation noise temperature. The general optical theorem will be proved in Sec.IV without regard to the sign of the noise temperature. Physical examples will be discussed in the concluding Sec.V.

II. A TRAVELING PLANE WAVE IN MATTER

Consider an electromagnetic plane wave

$$\mathbf{E} = \Re e \left[\mathbf{E}_0 e^{i(kz - \omega t)} \right] \quad (16)$$

traveling through a medium with a dielectric response function $\varepsilon(\omega + i0^+)$ so that

$$k = \frac{\omega}{c} \sqrt{\varepsilon(\omega + i0^+)}. \quad (17)$$

The intensity of the light beam described by the plane wave is proportional to

$$|\overline{\mathbf{E}}|^2 = \frac{1}{2} |\mathbf{E}_0|^2 \exp(-hz) \quad (18)$$

defines the extinction coefficient

$$h = 2\Im(k). \quad (19)$$

From Eq.(18) it is evident that

$$\begin{aligned} \text{an absorbing medium} &\implies (h > 0) \\ &\text{and} \\ \text{an amplifying medium} &\implies (h < 0). \end{aligned} \quad (20)$$

For a medium consisting of a dilute gas of polarizable particles for which $n|\alpha(\omega + i0^+)| \ll 1$, it follows from Eqs.(14), (17) and (19) that

$$h = n\sigma_{tot} = 4\pi n \left(\frac{\omega}{c} \right) \Im m \alpha(\omega + i0^+). \quad (21)$$

Eqs.(20) and (21) imply the optical Klein theorem:

$$\begin{aligned} \text{an absorbing medium} &\implies (\sigma_{tot} > 0) \\ &\text{and} \\ \text{an amplifying medium} &\implies (\sigma_{tot} < 0). \end{aligned} \quad (22)$$

The paradox of having $\sigma_{tot}(\omega \in \mathcal{B}) < 0$ in an amplifying frequency band \mathcal{B} has now been formally proved.

III. NOISE TEMPERATURE AND SPECTRAL FUNCTIONS

The quantum spectral functions for dipole moment $\mathbf{p}(t)$ fluctuations in a target may be defined by

$$S_{\pm}(\omega) = \frac{1}{3} \sum_{I,F} p_I |\langle F | \mathbf{p} | I \rangle|^2 \delta \left[\omega \mp \frac{(E_F - E_I)}{\hbar} \right], \quad (23)$$

where p_I is the target probability of being in an initial energy eigenstate $H|I\rangle = E_I|I\rangle$. Defining the free energy \mathcal{F} so that the initial probability is normalized to $\sum_I p_I = 1$, if the initial target is at temperature T ,

$$p_I(\text{Thermal}) = \exp\left(\frac{\mathcal{F} - E_I}{k_B T}\right) \quad (\text{Equilibrium}), \quad (24)$$

then the spectral functions in Eq.(23) obey the detailed balance condition

$$S_-(\omega) = S_+(\omega) \exp\left(-\frac{\hbar\omega}{k_B T}\right) \quad (\text{Equilibrium}). \quad (25)$$

The notion of having a noise temperature $T_n(\omega)$ which depends on frequency follows from the definition

$$S_-(\omega) = S_+(\omega) \exp\left[-\frac{\hbar\omega}{k_B T_n(\omega)}\right] \quad (\text{General}). \quad (26)$$

The Kubo formula for the electric polarizability of the target,

$$\alpha(\zeta) = \left(\frac{i}{3\hbar}\right) \int_0^\infty e^{i\zeta t} \langle \mathbf{p}(t) \cdot \mathbf{p}(0) - \mathbf{p}(0) \cdot \mathbf{p}(t) \rangle dt, \quad (27)$$

may be written in terms of the spectral functions in Eq.(23) employing

$$\langle \mathbf{p}(t) \cdot \mathbf{p}(0) - \mathbf{p}(0) \cdot \mathbf{p}(t) \rangle = 3 \int_{-\infty}^\infty e^{-i\omega t} [S_+(\omega) - S_-(\omega)] d\omega. \quad (28)$$

Eqs.(27) and (28) imply

$$\alpha(\zeta) = \frac{1}{\hbar} \int_{-\infty}^\infty \left[\frac{S_+(\omega) - S_-(\omega)}{\omega - \zeta} \right] d\omega, \quad (29)$$

from which one may deduce that

$$\Im m(\alpha(\omega + i0^+)) = \left(\frac{\pi}{\hbar}\right) [S_+(\omega) - S_-(\omega)]. \quad (30)$$

Eqs.(9), (26) and (30) imply

$$\sigma_{tot}(\omega) = \left(\frac{4\pi^2\omega}{\hbar c}\right) \left[1 - e^{-\hbar\omega/k_B T_n(\omega)}\right] S_+(\omega). \quad (31)$$

One may define the “symmetrical” noise spectral function according to $\bar{S} = (1/2)(S_+ + S_-)$. In terms of

$$\bar{S}(\omega) = \frac{1}{2} \left[1 + e^{-\hbar\omega/k_B T_n(\omega)}\right] S_+(\omega), \quad (32)$$

which in virtue of Eq.(23) obeys

$$\bar{S}(\omega) \geq 0, \quad (33)$$

Eq.(31) reads

$$\sigma_{tot}(\omega) = \left(\frac{8\pi^2\omega}{\hbar c}\right) \tanh\left[\frac{\hbar\omega}{k_B T_n(\omega)}\right] \bar{S}(\omega). \quad (34)$$

Eqs.(33) and (34) are the central results of this section. They allow Eq.(22) to be written in terms of the noise temperature as

$$\begin{aligned} (\text{absorbing}) \quad T_n(\omega) > 0 &\implies (\sigma_{tot}(\omega) > 0) \\ &\text{and} \\ (\text{amplifying}) \quad T_n(\omega) < 0 &\implies (\sigma_{tot}(\omega) < 0). \end{aligned} \quad (35)$$

Thus, the frequency band \mathcal{B} in Eq.(13) which is amplifying can be characterized as having a negative noise temperature $T_n(\omega \in \mathcal{B}) < 0$. As will be discussed in the concluding Sec.V, a variety of negative temperature systems[15, 16] can be thought to exhibit the electromagnetic Klein paradox.

IV. GENERALIZED OPTICAL THEOREM

In this section it is shown why the optical theorem of Eq.(8) holds true for an elastic scattering amplitude at positive or negative noise temperature. The negative noise temperature regime gives rise to a negative total cross section.

Suppose an electromagnetic wave is incident on a target localized near the origin of the spatial coordinate system. The electric field far from the target then has the familiar scattering form as $r \rightarrow \infty$; i.e.

$$\mathbf{E} \rightarrow \Re \left\{ E_0 e^{-i\omega t} \left(\mathbf{e}_i e^{i\omega z/c} + \mathbf{F} \cdot \mathbf{e}_i \frac{e^{i\omega r/c}}{r} \right) \right\}, \quad (36)$$

where \mathbf{F} is the dyadic elastic scattering amplitude

$$F_{i \rightarrow f} = \mathbf{e}_f^* \cdot \mathbf{F} \cdot \mathbf{e}_i. \quad (37)$$

The intensity of the incident wave in Eq.(36) is given by

$$I_0 = \left(\frac{c|E_0|^2}{8\pi} \right). \quad (38)$$

The intensity of the full wave is given by

$$I = \left(\frac{c|\mathbf{E}|^2}{4\pi} \right). \quad (39)$$

The optical theorem proof envisages detecting radiation directly behind the target but far away. We employ polar coordinates $\mathbf{r} = (x, y, z) = (\mathbf{r}_\perp, z)$ and the limits $z \rightarrow \infty$ so that $r_\perp \ll z$. The total cross section is then usually viewed in terms of the “missing intensity” detected on a screen placed directly behind but far away from the target. In mathematical terms, the total cross section may be defined as

$$\sigma_{total} = \lim_{z \rightarrow \infty} \int \left[\frac{I_0 - I(\mathbf{r}_\perp, z)}{I_0} \right] d^2 \mathbf{r}_\perp. \quad (40)$$

It is crucial to understand the physical meaning of Eq.(40). If the intensity behind the screen I is less than the incident intensity I_0 , i.e. if $I(\mathbf{r}_\perp, z \rightarrow \infty) < I_0$, then the total cross section is positive $\sigma_{tot} > 0$ which is most often presumed. The intensity behind the target is usually less than in the incident radiation because (i) the radiation is elastically scattered away at some angle with cross section σ_{el} or (ii) the radiation was inelastically absorbed by the target with cross section σ_{in} . Altogether, $\sigma_{tot} = \sigma_{el} + \sigma_{in}$ as in Eq.(10). But now consider Eq.(40) if the target is in an excited state. The target can be induced to decay by the incident radiation making the intensity on the behind the target screen bigger than the intensity of the incident radiation; i.e. $I(\mathbf{r}_\perp, z \rightarrow \infty) > I_0$ is a distinct possibility due to the *added radiation* from the decaying target. The excited target case $\sigma_{tot} < 0$ is thus quite possible.

In either of the above cases the optical theorem of Eq.(8) holds true as we shall now prove. From Eqs.(36)-(39) it follows that as $z \rightarrow \infty$ and as

$$r = \sqrt{z^2 + r_\perp^2} \rightarrow z + (r_\perp^2/2z) + \dots, \quad (41)$$

the intensity on the screen behind the target obeys

$$\left[\frac{I_0 - I(\mathbf{r}_\perp, z)}{I_0} \right] \rightarrow -\Re \left[\frac{2F_{i \rightarrow i} e^{i(\omega r_\perp^2/2cz)}}{z} \right], \quad (42)$$

which comes from the interference term when one takes the absolute square of the amplitude in Eq.(36). Now, Eqs.(40) and (42) imply the total cross section

$$\sigma_{tot} = - \lim_{z \rightarrow \infty} \Re \left\{ \int \left[\frac{2F_{i \rightarrow i} e^{i(\omega r_\perp^2/2cz)}}{z} \right] d^2 \mathbf{r}_\perp \right\}. \quad (43)$$

Using $\int (...) d^2 \mathbf{r}_\perp \rightarrow 2\pi \int_0^\infty (...) r_\perp dr_\perp$ allows for a simple evaluation of the integral in Eq.(43) leading to the optical theorem

$$\sigma_{tot} = \left(\frac{4\pi c}{\omega} \right) \Im m(F_{i \rightarrow i}). \quad (44)$$

It is important to note that the proof of the optical theorem given above in no manner invokes the sign of the total cross section. If the net intensity on the detectors directly behind the target is less than the incident intensity, then $\sigma_{tot} > 0$. If the net intensity on the detectors directly behind the target is more than the incident intensity, then $\sigma_{tot} < 0$. In either case Eq.(44) holds true.

V. CONCLUSIONS

We have shown that the electromagnetic Klein paradox arises in the form of a possible negative total cross section σ_{tot} for radiation to scatter off a target in an excited state. An excited state target can be viewed as having a negative noise temperature. Negative noise temperature targets are a reality[17, 18, 19] for laser or maser pumped materials.

Some of these ideas may be of basic interest and should be explored in detail for the strong interaction part of the standard model which invokes quantum chromodynamics (QCD) of quarks and gluons. At sufficiently small length scales, the effective interactions between the color charged particles within the plasma are thought to be weak. Short distance asymptotic freedom provides the perturbation theory basis for comparisons between QCD theory and scattering experiments. We have previously shown[31, 32, 33] that the asymptotically free vacuum has “negative dissipation” in the color dielectric response function $\epsilon(Q^2)$. The color dielectric response function determines the strong interaction coupling strength via $\alpha_{QCD}(Q^2) = g^2/\{4\pi\hbar c\epsilon_{QCD}(Q^2)\}$. Negative dissipation implies an asymptotically free QCD excited state amplifier unstable to decay.

The fact that the Landau ghost does not haunt QCD perturbation theory is totally dependent upon the existence of negative color dissipation. It has been shown in our earlier work that while asymptotic freedom exorcises the Landau ghost from QCD, it ladens it with negative dissipation. A direct consequence of confinement and asymptotic freedom in QCD is the notion that an applied color electric field produces an “anti-Ohm’s law” color current which then cools the perturbative QCD vacuum.

Physical negative dissipation can occur when matter is pumped up by an external energy source. The resulting “amplifier” excited state can then return the energy when the matter decays back into the true ground state.

The physical interpretation of vacuum cooling induced by the application of a high frequency chromo-dynamic field is (at first glance) more than a little obscure. How might one cool what is already asserted to be the vacuum? A true quantum vacuum is conventionally visualized as having the lowest possible energy. No further cooling should then be possible. Any notion of asymptotic freedom as a justification of QCD perturbation theory must then refer to an excited state color current amplifier with negative resistance. One may indeed cool down an excited state. We have then been forced to conclude that the asymptotically free QCD vacuum is in reality unstable which must eventually decay into the true vacuum. But the situation with regard to comparisons between theory and experiment needs further elucidation.

Experimental achievement of negative noise temperatures, characteristic of electronic energy levels with inverted populations, was absolutely essential for the construction of the original maser and laser devices. Negative noise temperatures also occur naturally within some matter clouding galaxies. The negative dissipative part of the electrical conductivity in some astrophysical matter is made manifest via the measured negative absorption (i.e. amplification) of the electromagnetic radiation that passes through these materials. However, we must also realize that amplification in excited state systems can only have a finite lifetime. The excited state atoms decay into their ground states after giving rise to a burst of electromagnetic radiation. To revert again to the excited state amplifier status, energy must be pumped back into the matter by an external source. The instability of the QCD vacuum needs to be investigated in depth in view of the results of the present paper.

A negative electromagnetic cross section simply means there is more radiation energy behind the target in the outgoing state than there would be if the target were *not* present. The target then represents an electromagnetic amplifier. The amplifier must be pumped into an excited state by an external energy source before the next incident wave is sent to scatter off the target. Such amplifiers have been of considerable interest in astrophysics[20, 21, 22, 23, 24] where they occur naturally in interstellar media.

The relativistic Klein paradox does not in reality imply any sort of negative probability. In the Dirac theory, one may create (from pair production) more electrons in the final state than were present in the initial state. But in probability terms the *total charge* would remain conserved[25, 26, 27, 28, 29, 30]. Similarly, in the Maxwell theory, one may create (from atomic quantum state decays) more photons in the final state than were present in the initial state. But in probability terms the *total energy* would remain conserved. From the above optical theorem proof, the total cross section for the scattering of electromagnetic radiation may obey $\sigma_{tot}(\omega \in \mathcal{B}) < 0$ because the radiation energy in the final state may exceed (by far) the radiation in the initial state, but the probability for the scattering event is nevertheless positive. In this manner, the electromagnetic Klein paradox may be resolved.

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